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Abstract

Periodic signals are those which repeat itself after certain interval of time. The time after which it repeats is called as the period of the signal. Periodic signals are used almost everywhere in application of electrical engineering such as, supply of electricity means supplying AC voltage constantly. This supply of ac is periodic waveform [1]. Continuous time sinusoids are periodic if they are not truncated and period of this type of signal can be predicted in advanced just comparing the angular frequency (ω). Whenever addition or multiplications of two or more sinusoids or any other continuous time signals are involved then it is not that simple to find the period of the resulting signal [2]. All the cases of addition and multiplication of different types of continuous time signals were taken into account and the result were obtained from MATLAB. Findings were analyzed and generalized being based on the result obtained from MATLAB. When addition of two or more continuous time signals takes place, the period of resulting signal is the Least Common Multiple (LCM) of the period of individual signal. When multiplication of two or more continuous time signal are involved the period of resulting signal is the half the value of Least Common Multiple (LCM) of the period of individual signal, if any one of the signal has both positive and negative amplitude. Else the period of the resulting signal is Least Common Multiple of the period of individual signal. When addition of two or more continuous time signal are involved the period of resulting signal is the value of Least Common Multiple (LCM) of the period of individual signal, even if any one of the signal has both positive and negative amplitude. Generally Continuous time Fourier series is performed to obtain the spectrum either that may be magnitude spectra or phase spectra. Periodicity and the period of the signal are vital for representing time domain signal to frequency domain.

Keywords: Periodic waveform; both positive and negative amplitude; Fourier series; continuous time signals; LCM.

1. Introduction

Periodic signal is that type of signal which has a definite pattern and repeats over and over with time or period of T. In continuous time if the transformed signal is same as x(t), then the signal is periodic. That is for periodic signal.

\[ x(t) = x(t + T) \]

\[ T = 1, 2, 3, \ldots \]

The smallest value of period T which satisfies above equation is called the fundamental period To. In the Figure 1, the fundamental period is either T2 or 2T1. If \( x(t) = e^{i\omega t} \) and periodic then according to the definition of periodicity \( x(t) = x(t + T) \)

Figure 1. Continuous time period signal.

\[
e^{j\omega t} = e^{j\omega(t+T)} = e^{j\omega t}e^{j\omega T}
\]

\[
e^{j\omega T} = 1
\]

\[
wT = 2\pi
\]

\[
\therefore \quad T = \frac{2\pi}{\omega}
\]

If \(w=0\), then \(x(t) = 1\). This implies a constant value thus, the function is periodic for any value of \(T\) which is insignificant in case.

2. Methodology and Simulation

2.1. Addition of two signals

The Addition of two periodic signals and the period of the resulting signal which is shown in Figure 1. Let us take two periodic signals and the third signal as a sum of these signals.

Let us use MATLAB for the illustration

Source code:

\[
t=0:0.001:1000
\]
\( a = \cos\left(\frac{t}{7} + \frac{\pi}{4}\right) \)
\( b = \cos\left(\frac{t}{11} + \frac{\pi}{30}\right) \)
\( c = a + b \)

```matlab
subplot(311)
pplot(t,a)
grid on
subplot(312)
pplot(t,b)
grid on
subplot(313)
pplot(t,c)
grid on
```

**Figure 4.** Multiplication Simulation results of two periodic signal and sum of them by using MATLAB.

Looking at the MATLAB’s output, we see the period of signal \( a \) is \( 14\pi \), the period of the signal \( b \) is \( 22\pi \) and the period of the resulting signal \( c \) is LCM of \( a \) and \( b \) i.e. \( 154\pi \).

### 2.2. Multiplication of two signals

Multiplication of two periodic signal and period of the resulting signal is shown in Figure 3. Let us take two periodic signals and the third signal as a product.

Let us take two periodic sinusoidal signals \( a \) and \( b \) and the third signal \( c \) as a product of these signals. By using MATLAB, we can have the following results as shown in Figure 54.

Source code:
Figure 5. Multiplication of two periodic signal and period of the resulting signal.

t=0:0.001:1000
a= cos((t/7)+(pi/4))
b= cos((t/11) + (pi/30))
c= a.*b
subplot(311)
plot(t,a)
grid on
subplot(312)
plot(t,b)
grid on
subplot(313)
plot(t,c)
grid on

Figure 6. Multiplication Simulation results of two periodic signal and sum of them by using MATLAB.
Looking at the MATLAB’s output, we see the period of signal a is $14\pi$, the period of the signal b is $22\pi$ and the period of the resulting signal c is half the LCM of a and b i.e. $\frac{77\pi}{2}$.

### 3. Findings

Addition of two or more continuous time periodic signals always yields periodic signal. Addition of two or more continuous time signals will not be periodic even if one of them is not periodic. Multiplication of two or more continuous time periodic signals always yields periodic signal. Multiplication of two or more continuous time periodic signals will not be periodic even if one of them is not periodic. When addition of two or more continuous time signals takes place, the period of resulting signal is the Least Common Multiple (LCM) of the period of individual signal. When multiplication of two or more continuous time signal are involved the period of resulting signal is the half the value of Least Common Multiple (LCM) of the period of individual signal, if any one of the signal has both positive and negative amplitude. Else the period of the resulting signal is Least Common Multiple of the period of individual signal. When addition of two or more continuous time signal are involved the period of resulting signal is the value of Least Common Multiple (LCM) of the period of individual signal, even if any one of the signal has both positive and negative amplitude. 3.2 Antenna:

### 4. Applications

A periodic signal is a signal that repeats itself over a fixed period of time such as sinusoidal, square, and triangular or saw tooth waveform. So, basically they are used in almost every application of electrical engineering. Figure 7 and 8 shows the periodic sine wave in 12 and 6 Hz respectively.

![Figure 7. Periodic Sinewave of 12 Hz.](image1)

![Figure 8. Periodic Sinewave of 6 Hz.](image2)
Periodic waveforms are also responsible for driving oscillators which is very important in computer applications where a CPU may need to operate according to the clock speed that is determined by the oscillator [3]. It is always important to know the period of the periodic signal to predict clock frequency in case of oscillator and to know the frequency of electrical supply used in either a tube light or any ac bulb which are used in daily life.

The power we use at home has a frequency of 50 or 60 Hz. The period of this sine wave can be determined as follows:

For 60 Hz:
\[ T = \frac{1}{f} = \frac{1}{60} = 0.0166 \text{ s} = 0.0166 \times 10^3 \text{ ms} = 16.6 \text{ ms} \] [4]

For 50 Hz:
\[ T = \frac{1}{f} = \frac{1}{50} = 0.02 \text{ s} = 0.02 \times 10^3 \text{ ms} = 20 \text{ ms} \]

5. Conclusion

Periodic signals are those which repeat itself after certain interval of time. The time after which it repeats is called as the period of the signal. Periodic signals are used almost everywhere in application of electrical engineering such as, supply of electricity means supplying AC voltage constantly. This supply of ac is periodic waveform. Continuous time sinusoids are periodic if they are not truncated and period of this type of signal can be predicted in advanced just comparing the angular frequency (\(\omega\)). Whenever addition or multiplications of two or more sinusoids or any other continuous time signals are involved then it is not that simple to find the period of the resulting signal. All the cases of addition and multiplication of different types of continuous time signals were taken into account and the result were obtained from MATLAB. Findings were analyzed based on the result obtained from MATLAB.

References